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INSTABILITY OF SCALAR PERTURBATIONS IN A PHANTOMIC COSMOLOGICAL SCENARIO

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Abstract

Scalar perturbations can grow during a phantomic cosmological phase as the big rip is approached, in spite of the high accelerated expansion regime, if the equation of state is such that $\frac{p}{\rho} = \alpha < -\frac{5}{3}$. It is shown that such result is independent of the spatial curvature. The perturbed equations are exactly solved for any value of the curvature parameter k and of the equation of state parameter α. Growing modes are found asymptotically under the condition $\alpha < -\frac{5}{3}$. Since the Hubble radius decreases in a phantom universe, such result indicates that a phantom scenario may not survive longtime due to gravitational instability.

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In the ordinary theory of cosmological perturbation [1, 2, 3] there are two fundamental regimes defined from the notion of *Jean's length*, λ_J. Perturbations whose scales are such that $\lambda > \lambda_J$ suffer gravitational instability since the gravitational attraction dominates over the pressure reaction to contraction; on the other hand, perturbations whose scales satisfy the condition $\lambda < \lambda_J$, tends to oscillates due to the effectiveness of the pressure opposition to the gravitational collapse. In the relativistic version of the theory of cosmological perturbation we must add a new relevant scale, given by the Hubble radius. The Hubble radius define, in some sense, the effective causal region, and consequently the region where the effects of the microphysics phenomenon (responsible for the pressure) may play a relevant rôle in the process of gravitational collapse. Perturbations whose scales are much large than the Hubble radius tends to become frozen, while for those smaller than the Hubble radius, the pressure tends to produce a damping effect. Moreover, if the spatial section of the metric representing the universe is not flat a new scale appears which is connected to the curvature parameter.

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In cosmology, concomitant with the interplay between the gravitational attraction and the pressure resistance, there is a supplementary relevant effect due to the expansion of the universe, which acts as a friction term in the fundamental equations of the perturbed quantities: the expansion leads to a damping in the evolution of perturbations. In general, for very large perturbations, that are connected with the large structures existing in the universe, there are two modes: a growing or a constant mode and a decreasing mode. However, if the pressure is negative, and the expansion becomes accelerated, gravity may become repulsive and we generally find only decreasing modes: the perturbations are always damped and structures can not be formed.

There are quite strong evidences that the universe today is in an accelerated expansion phase [4]. If this is true, the material content of the universe must be dominated by an exotic fluid whose pressure is negative. There are many candidates for this exotic fluid, like cosmological constant, quintessence, K-essence, Chaplygin gas, etc., each of them having its advantages and disadvantages from the theoretical point of view. We address the reader to recent reviews on the many dark energy models existing in the literature [5, 6]. Such negative pressure fluid may generate repulsive effects driving the accelerated expansion. There are some claims that the observational data favors a phantom fluid, that is, a fluid whose negative pressure violates the dominant energy condition $p + \rho \geq 0$ [7]. Representing such fluid by a self-interacting scalar field, a phantom fluid requires a "wrong" sign in the kinetic term. This may imply instability at quantum level. However, interesting propositions have been made in what concerns such quite exotic fluid, mainly in the context of ghost condensation existing in some string configurations [9].

Classically, one of the most striking feature of phantom fluids is the fact that its energy density grows with the expansion. This is consequence of the violation of the strong energy condition, and it may lead to a future singularity in a finite future time, the so-called *big rip*. This is, of course, a very undesirable feature. But it has been shown [8] that in a single fluid approximation, for a spatially flat universe, the scalar perturbations can grow when the scales of the perturbation are greater than the Hubble radius. Since, the Hubble radius decreases, for a phantom dominated universe, the isotropy and homogenous condition would not be satisfied anymore as the big rip is approached, leading perhaps to the avoidance of this future singularity, leaving a very inhomogeneous universe. This situation may occur if the pressure is negative enough in order to satisfy the condition $\frac{p}{\rho} = \alpha < -\frac{5}{3}$.

In the present work we extend those result showing that phenomenon of enhancing of the inhomogeneities in large scales is independent of the spatial curvature, and that the critical point $\alpha = -\frac{5}{3}$ is present in any class of homogeneous and isotropic universe. In order to do so, we will solve the perturbed equations for scalar modes for any value of k and α. An asymptotic analysis will reveal the existence of critical behavior associated to $\alpha = -\frac{5}{3}$.

For a universe dominated by a fluid whose equation of state is given by

$p = \alpha\rho$, the relevant equation is

$$\frac{a'^2}{a^2} + k = \frac{8\pi G}{3}\rho\, a^2 \quad , \quad \rho = \rho_0\, a^{-3(1+\alpha)} \quad . \tag{1}$$

The primes indicate derivations with respect to the conformal time η, and k is the spatial curvature parameter, $k = \pm 1, 0$. The solution for this equation may be written in a unified form as

$$a(\eta) = a_0 \left[\frac{1}{\sqrt{k}} \sin\left(\frac{1+3\alpha}{2}\sqrt{k}\eta\right)\right]^{\frac{2}{1+3\alpha}} \quad . \tag{2}$$

For $k = 1$, this solution represents a universe that begins and end at a singularity at $a = 0$ for $1 \leq \alpha < -\frac{1}{3}$, while for $\alpha < -\frac{1}{3}$ it represents a bouncing universe. On the other hand, for $k = -1, 0$, it represents an ever expanding universe for any value of α, with the following characteristics: for $1 \geq \alpha > -\frac{1}{3}$, the expansion implies $0 \leq \eta < \infty$ corresponding, in terms of the cosmic time t, to $0 \leq t < \infty$; for $-\frac{1}{3} > \alpha$, the expansion implies $-\infty < \eta \leq 0$, corresponding to $0 \leq t < \infty$ if $-\frac{1}{3} > \alpha \geq -1$ and $-\infty < t \leq 0$ if $-1 > \alpha$. This last feature leads to the notion of big rip.

To study the evolution of scalar perturbations, we use the gauge invariant formalism. For a perfect fluid the evolution of the scalar perturbation is given by a single equation for the gravitational potential Φ [3, 10]:

$$\Phi'' + 3(1+\alpha)H\Phi' + \left\{\alpha\, n^2 + 2H' + (1+3\alpha)(H^2 - k)\right\}\Phi = 0 \quad , \tag{3}$$

where $H = \frac{a'}{a}$ and n^2 is the eigenvalue of the Laplacian operator $\nabla^2\Phi = -n^2\Phi$. The perturbed equation can be recast under the following form

$$(1-z)z\Phi'' + \frac{7+9\alpha}{2(1+3\alpha)}(1-2z)\Phi' + \tilde{n}^2\Phi = 0 \quad , \tag{4}$$

where

$$z = \frac{1+\cos(\sqrt{k}\theta)}{2} \quad , \quad \tilde{n}^2 = \frac{4k}{(1+3\alpha)^2}\left[\alpha n^2 - 2(1+3\alpha)k\right] \quad , \quad \theta = \frac{1+3\alpha}{2}\eta \quad . \tag{5}$$

The solution of (4) can be represented under the form of hypergeometric functions. It reads in general, for any value of k and α, as follows:

$$\Phi_n(\eta) = c\, {}_2F_1\left[A_+, A_-; B; z\right] + \bar{c}\, z^{1-B}\, {}_2F_1\left[A_+ - B + 1, A_- - B + 1; 2 - B; z\right] \quad , \tag{6}$$

where

$$A_\pm = \frac{1}{2}\left\{6\frac{1+\alpha}{1+3\alpha} \pm \sqrt{36\frac{(1+\alpha)^2}{(1+3\alpha)^2} + 4\tilde{n}^2}\right\} \quad , \quad B = \frac{7+9\alpha}{2(1+3\alpha)} \quad , \tag{7}$$

and $c, \bar{c}$ are constants.

The asymptotic analysis for the flat case $k = 0$ was made in reference [8]. In this case, the hypergeometric's function reduces to Bessel's functions. It has been shown that one of the two modes remains constant, in the long wavelength limit, for any value of α. In the same limit, however, the other mode decreases when $\alpha > -\frac{5}{3}$, but grows with time when $\alpha < -\frac{5}{3}$. For $\alpha = -\frac{5}{3}$, both modes are constants. In the case $k \neq 0$, such analysis, in terms of large or small wavelength limit, is more involved since, contrarily to the flat case, we have a scale given by the Hubble radius and another scale given by the curvature. It becomes easier, in this sense, and for our purpose more relevant, to consider the behavior in the extremes of the time interval. In order to perform this analysis, we must take into account some convenient transformation properties of the hypergeometric functions. In special, the following transformations will be useful (see reference [11]):

$$
\begin{aligned}
{}_2F_1(A,B;C;z) = \frac{\Gamma(C)\Gamma(C-A-B)}{\Gamma(C-A)\Gamma(C-B)}{}_2F_1(A,B;A+B-C+1;1-z) \quad + \\
(1-z)^{C-A-B}\frac{\Gamma(C)\Gamma(A+B-C)}{\Gamma(A)\Gamma(B)}{}_2F_1(C-A,C-B;C-A-B+1;1-z) \qquad ;(8)
\end{aligned}
$$

$$
\begin{aligned}
{}_2F_1(A,B;C;z) = \frac{\Gamma(C)\Gamma(B-A)}{\Gamma(B)\Gamma(C-A)}(-1)^A z^{-A}{}_2F_1\left(A,A+1-C;A+1-B;\frac{1}{z}\right) \quad + \\
\frac{\Gamma(C)\Gamma(A-B)}{\Gamma(A)\Gamma(C-B)}(-1)^B z^{-B}{}_2F_1\left(B,B+1-C;B+1-A;\frac{1}{z}\right) \qquad .(9)
\end{aligned}
$$

Hence, we have the following asymptotic behaviors:

$$
z \to 0 \quad \Rightarrow \quad {}_2F_1(A,B;C;z) \sim z^{1-C} \quad , \tag{10}
$$

$$
z \to 1 \quad \Rightarrow \quad {}_2F_1(A,B;C;z) \sim \frac{\Gamma(C)\Gamma(C-A-B)}{\Gamma(C-A)\Gamma(C-B)}(1-z)^{C-A-B} + \frac{\Gamma(C)\Gamma(A+B-C)}{\Gamma(A)\Gamma(B)} \quad , \tag{11}
$$

$$
z \to \infty \quad \Rightarrow \quad {}_2F_1(A,B;C;z) \sim \frac{\Gamma(C)\Gamma(B-A)}{\Gamma(B)\Gamma(C-B)}(-1)^A\, z^{-B} + \frac{\Gamma(C)\Gamma(A-B)}{\Gamma(A)\Gamma(C-B)}(-1)^B\, z^{-A} \quad . \tag{12}
$$

Using these expressions, we can determine the behavior of the perturbations in the two different extremities of the time interval, for each value of k.

- $k = 1$. In this case the conformal time interval is $0 \leq \eta \leq \frac{2\pi}{1+3\alpha}$ for $\alpha > -\frac{1}{3}$ and $\frac{2\pi}{1+3\alpha} \leq \eta \leq 0$ for $\alpha < -\frac{1}{3}$. Using the asymptotic expressions written above, we find the following behaviors: for $\alpha > -\frac{1}{3}$, there is initially two decreasing modes and, as the universe approaches the big crunch at $\eta = \frac{2\pi}{1+3\alpha}$, there are a constant mode and a growing mode; for $-\frac{1}{3} > \alpha > -\frac{5}{3}$ there is initially, during the contraction phase, a growing

mode and a constant mode, and as the scale factor diverges in the other asymptotic, there is a constant mode and a decreasing mode; for $\alpha = -\frac{5}{3}$, both modes are constant at the beginning of the contraction phase and at the end of the expansion phase; for $\alpha < -\frac{5}{3}$, there is a constant mode and a decreasing mode at the universe begins to contract, and there are two increasing modes as the universe approaches the big rip.

- $k = -1$. The range of the conformal time is $0 \leq \eta < \infty$ for $\alpha < -\frac{1}{3}$ and $-\infty < \eta \leq 0$ for $\alpha < -\frac{1}{3}$. The open case is more involved because, in opposition to the closed universe, the asymptotic behavior of the modes depends on the scale of the perturbation. Let us consider the situation where the eigenvalues of the Laplacian operator is null. Hence, when $\alpha > -\frac{1}{3}$ there is initially and in future infinity two decreasing modes. For $-\frac{1}{3} < \alpha < -\frac{5}{3}$ there is initially two decreasing modes, but in the future infinity there is a constant mode besides a decreasing mode; the case $\alpha = -\frac{5}{3}$ differs from the preceding one by the fact that in future infinity both modes are constant. Finally, when $\alpha < -\frac{5}{3}$, both modes are initially decreasing but they become growing modes as the big rip is approached.

When $\alpha = -\frac{1}{3}$, in all cases, the same features observed in the flat universe are reproduced here, since for this particular equation of state the matter density scales as the curvature parameter in the Friedmann's equation.

The main conclusion of the previous analysis is that a phantom cosmological scenario is highly unstable against scalar perturbations under the condition of an isotropic and homogeneous background universe if the pressure is negative enough, that is $\frac{p}{\rho} < -\frac{5}{3}$: the scalar perturbations grows as the big rip is approached. As in the flat case, the Hubble radius shrinks with time in the phantomic case. This means that the large scale approximation becomes essentially valid asymptotically for all perturbation scales in the phantomic case. The analysis was made using a perfect fluid material content. However, at large scales, it is expected that a more fundamental representation, using for example, scalar fields, must give the same results as the perfect fluid case [12].

A curious feature of the above results concerns the critical point $\alpha = -\frac{5}{3}$. As far as we know, it does not correspond to any energy condition (contrarily to $\alpha = -\frac{1}{3}$ and $\alpha = -1$). Also, it seems not related neither to the Hubble parameter, nor to deceleration parameter, or even to the statefinder parameters [13]. However, it seems to be a general critical point for the analysis of perturbation, that is not reflected in the kinematical quantities like those we have quoted above. A dynamical system analysis of the background does not reveal any particular feature at $\alpha = -\frac{5}{3}$ [14]: it is not a critical point for the background. Moreover, in the perturbed equation (3) there is no explicit special structure for that particular value of the parameter α. The nature of this critical point must still be cleared up.

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